ESTIMATION OF THE DEPRECIATION RATE OF PHYSICAL AND R&D CAPITAL IN THE U.S. TOTAL MANUFACTURING SECTOR

M. ISHAQ NADIRI and INGMAR R. PRUCHA*

Numerous studies on production and cost, the sources of productivity, and endogenous growth have recognized the pivotal role played by physical and R&D capital stocks. Analysis of the contributions of these capitals often requires measures of the stocks of physical and R&D capital, which in turn requires measuring their depreciation rates. In this paper we specify a model of factor demand that allows us to estimate the depreciation rates of both physical and R&D capital jointly with other model parameters. The model is estimated for the U.S. total manufacturing sector.

I. INTRODUCTION

Numerous studies on production and cost, the sources of productivity, and endogenous growth have recognized the pivotal role of the physical capital stock. Studies at various levels of aggregation over assets and industries have emphasized the crucial role of investment in plant and equipment in the growth of demand and productive capacity. It is clearly recognized by economists and policymakers that knowledge capital, approximated by R&D capital, is crucial for productivity growth and the transformation of the industrial structure of an economy.

The distinction between net and replacement investment in these types of capital is important for policy purposes. Critical to an analysis of the contributions of physical and R&D capital is an accurate measurement of the stocks of physical and R&D capital, which in turn requires measuring their depreciation rates. However, measuring these depreciation rates provides a formidable challenge. In the following we present an approach that uses an econometric model and gross investment data to generate estimates of the depreciation rates and consistent series for the stocks of physical and R&D capital as well.

The conventional procedure for estimating the stock of physical capital is based on the perpetual inventory method. Unfortunately, the assumptions underlying this procedure are typically not subjugated to formal statistical testing. So far only a few econometric studies provide estimates for the depreciation rate of physical capital within the context of a behavioral model.[1] Very little effort has been made, except for Pakes and Schankerman [1978; 1986], to measure the depreciation rates of the stock of R&D. Researchers doing applied work typically assume an arbitrary depreciation rate of 0.10 to 0.15 percent to construct the stock of R&D capital using the perpetual inventory method.

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1. For a discussion of measurement issues and the perpetual inventory method see, e.g., the papers by Hulten [1991], Jorgenson [1991] and Triplett [1992]. For some econometric studies see Bischoff and Kokkelenberg [1987], Epstein and Denny [1980], Kollintzas and Choi [1985], Prucha and Nadiri [1991], and the paper by Doms in this issue.

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In this paper we estimate the depreciation rates of both physical and R&D capital stocks for the U.S. total manufacturing sector within the framework of a factor demand model. We also generate, using our analytical framework, "capital" stock series for both types of capital, which are consistent with the estimated depreciation rates. We need only gross investment data to obtain the implied estimates of the depreciation rates and to generate consistent capital stock series.

The model considered here is a special case of the theoretical model in Prucha and Nadiri [1991], which allows for the estimation of variable depreciation rates for several types of capital stocks. In that modeling framework the firm is allowed to combine its beginning-of-period stocks of physical and R&D capital with other inputs to produce output as well as end-of-period stocks of both types of capital. Basic elements of that modeling framework date back to Hicks [1946] and Malinvaud [1953] and were recently discussed by Diewert [1977; 1980]. In the literature on dynamic demand models such an approach was first adopted by Epstein and Denny [1980] and more recently by Kollintzas and Choi [1985] for a single capital good, and on a theoretical level in Bernstein and Nadiri [1987a; 1987b].

The paper is organized as follows. The specification of the model is presented in section II. Section III is devoted to a discussion of the data and to the presentation of the parameter estimates of the model. In section IV we present the empirical results for the depreciation rates for physical and R&D capital and compare them with those reported in the literature. We compare the physical capital stock estimates generated by our model with the "official" capital stock estimates produced by U.S. Department of Commerce, Bureau of Economic Analysis. In this section we also present our estimates of the decomposition of gross investment into replacement and expansion investment for both physical and R&D capital. These decompositions are important from the vantage point of public policy analysis. The last section of the paper contains our conclusions and suggestions for certain extensions of the model and results.

II. MODEL SPECIFICATION

We assume that the technology can be represented by a factor requirement function of the form

$$L_t = L(Y_t, M_t, K_{t-1}, R_{t-1}) \tag{1}$$

where L_t and M_t denote, respectively, labor input and material input; K_t and R_t denote, respectively, the end-of-period stocks of physical and R&D capital; and Y_t denotes gross output. Labor and materials are taken to be variable inputs. As in many studies the stock of R&D may be considered to represent a technological index that shifts the production frontier.[2] Alternatively, given the technology satisfies appropriate curvature restrictions, the stock of R&D may be viewed as a factor input. The capital stocks K_t and R_t accumulate according to the following equations:

$$K_t = =I_t^K + (1 - \delta^K)K_{t-1}, \tag{2}$$

$$R_t = I_t^R + (1 - \delta^R)R_{t-1}, \tag{3}$$

where I_t^K and I_t^R denote gross investment in physical and R&D capital, and δ^K and

2. Compare, e.g., Romer [1990]. The high correlation between the stock of R&D and time precluded the introduction of time as another exogenous shift variable. Clearly, future research should explore the relative contributions of R&D and exogenous technical change to growth in more detail, possibly based on a more disaggregated data set and a richer model specification.

δ^R denote the depreciation rates of physical and R&D capital, respectively.

The first-order conditions for minimizing labor and material costs for given stocks of physical and R&D capital are given by

$$\partial L_t / \partial M_t + p_t^M = 0, \tag{4}$$

where p_t^M is the price of materials normalized by the price of labor, both of which are assumed to be exogenous. Let $\hat{M}_t$ denote the minimizing value for materials, then the technology can be represented equivalently in terms of the following normalized restricted cost function:

$$\begin{aligned} G_t &= G(p_t^M, K_{t-1}, R_{t-1}, Y_t) \\ &= L(Y_t, \hat{M}_t, K_{t-1}, R_{t-1}) + p_t^M \hat{M}_t. \end{aligned} \tag{5}$$

The function $G(\cdot)$ is assumed to be homogeneous of degree zero in p^M, nondecreasing in Y, nonincreasing in K and R, and concave in p^M. We also assume that $G(\cdot)$ is convex in K. A corresponding assumption is not maintained a priori with respect to R; thus in our model the stock of R&D may simply serve the role of a technological index that shifts the variable cost function. If $G(\cdot)$ is convex in both physical and R&D capital (K and R) then, as indicated above, the stock of R&D can also be interpreted as a factor input.

For the empirical analysis we adopt (dropping subscripts t) the following functional form:[3]

$$\begin{aligned} G(p^M, K_{-1}, R_{-1}, Y) = \\ Y\,\{\alpha_0 + \alpha_M p^M + 0.5\alpha_{MM}(p^M)^2\} \\ + \alpha_K K_{-1} + \alpha_R R_{-1} + \alpha_{KM} K_{-1} p^M + \alpha_{RM} R_{-1} p^M \\ + \{0.5\alpha_{KK} K_{-1}^2 + \alpha_{KR} K_{-1} R_{-1} \\ + 0.5\alpha_{RR} R_{-1}^2\}\,/\,Y. \end{aligned} \tag{6}$$

The above functional form imposes constant returns to scale on the technology. The convexity of the cost function $G(\cdot)$ in physical capital K and the concavity in p^M, the price of materials, implies that $\alpha_{KK} > 0$ and $\alpha_{MM} < 0$. If $G(\cdot)$ is convex in both physical and R&D capital (K and R) then we have also $\alpha_{RR} > 0$ and $\alpha_{KK}\alpha_{RR} - \alpha_{KR}^2 > 0$.

The demand equations for labor and materials can be derived via Shephard's lemma, i.e., as $L_t = G_t - p_t^M M_t$ and $M_t = \partial GH_t / \partial p_t^M$. Let $\hat{p}_t^M$ and $\hat{Y}_t$ denote the expected values for the price of materials and the level of output, then (6) implies the following demand equations for labor and material inputs conditional on those expected values:

$$\begin{aligned} L_t = \{\alpha_0 - 0.5\alpha_{MM}(\hat{p}_t^M)^2\}\hat{Y}_t \\ + \alpha_K K_{t-1} + \alpha_R R_{t-1} + \\ \{0.5\alpha_{Kk} K_{t-1}^2 + \alpha_{KR} K_{t-1} R_{t-1} \\ + 0.5\alpha_{RR} R_{t-1}^2\}\,/\,\hat{Y}_t, \end{aligned} \tag{7}$$

3. This functional form for a normalized variable cost function was introduced by Denny, Fuss and Waverman [1981] and Morrison and Berndt [1981]. It represents a second-order approximation (in levels) to a constant-returns-to-scale technology; Nadiri and Prucha [1983; 1990b] generalized this functional form to the case of homothetic technologies. We note that the normalized variable cost function obtained by choosing the price of materials rather than the price of labor as the numeraire would represent an alternative form of the technology. We have also experimented empirically with this alternative functional form, but found that when we modeled the technology with this alternative functional form some of the estimated parameters violate theoretical restrictions; hence we do not report corresponding results here. We note, however, that the estimates for the depreciation rates of physical capital and R&D turned out to be similar to those reported here.

$$(8)\qquad M_t + \{\alpha_M + \alpha_{MM}\hat{p}_t^M\}\hat{Y}_t + \alpha_{KM}K_{t-1} + \alpha_{RM}R_{t-1}.$$

As discussed above, one of our objectives is to obtain estimates for the depreciation rates of physical and R&D capital. Given that the values of the depreciation rates are not assumed a priori, the stocks of physical and R&D capital are unobserved. From equations (2) and (3) we see that the stock of capital and R&D can be expressed as a function of, respectively: past gross investments I_t^K, I_{t-1}^K,..., and I_t^R, I_{t-1}^R,...; the initial stocks K_0 and R_0; and the depreciation rates δ^K and δ^R, i.e.,

$$K_t = \sum_{i=0}^{t-1}(1-\delta^K)^i I_{t-i}^K + (1-\delta^K)^t K_0$$

and

$$R_t = \sum_{i=0}^{t-1}(1-\delta^R)^i I_{t-i}^R + (1-\delta^R)^t R_0.$$

In principle, we can now substitute those expressions into (7) and (8), which yields a system of equations of the form

$$(7')\qquad L_t = L_t(I_t^K, I_{t-1}^K,\ldots, I_1^K, I_t^R, I_{t-1}^R, \ldots, I_1^R, K_0, R_0, \hat{p}_t^M, \hat{Y}_t, \alpha, \delta^K, \delta^R),$$

$$(8')\qquad M_t = M_t(I_t^K, I_{t-1}^K,\ldots, I_1^K, I_t^R, I_{t-1}^R, \ldots, I_1^R, K_0, R_0, \hat{p}_t^M, \hat{Y}_t, \alpha, \delta^K, \delta^R),$$

where α represents the vector of parameters that characterize the normalized variable cost function. If observations on the initial stocks are available, then in the system of equations so obtained *all variables are observable*, but the depreciation rates, δ^K and δ^R, are additional model parameters that need to be estimated together with the other model parameters α.[4] For purposes of estimation we also add stochastic disturbance terms to each of the factor demand equations in (7) and (8) or (7′) and (8′), and also allow for autocorrelation in those disturbances.

For the actual numerical computation of estimators of the model parameters it may be inconvenient to explicitly program the substituted expression on the right-hand side of (7′) and (8′). Numerical algorithms for the computation of estimators that are defined as optimizers of some statistical objective function generally require the numerical evaluation of the statistical objective function for different sets of parameter values. This in turn requires the computation of the residuals for the behavioral equations for different sets of model parameters. For any given set of values for the depreciation rates of physical and R&D capital we can solve (2) and (3) numerically for the stocks of capital, K_t and R_r, in a recursive manner. Using the obtained values for the capital stocks we can then compute for a given set of model parameters the corresponding residuals directly from (7) and (8). Hence, rather than to work with the substituted equations (7′) and (8′) we can, in evaluating the statistical objective function, first solve (2) and (3) numerically in a recursive manner and then use the numerical solution for K_t and R_t in the evaluation of the objective function based on (7) and (8).

The statistical objective function underlying the parameter estimates reported in the next section is the Gaussian full information maximum likelihood (FIML) func-

4. If observations on the initial stocks are not available, then the initial stocks could be treated as further unknown parameters. An extension of the model would be to allow the depreciation rate to be a decision variable for the firm. Such an extension is discussed in Prucha and Nadiri [1991], where the depreciation rate is expressed as a function of relative prices and output; see also the corresponding discussion in the introduction.

tion. We used the subroutine VA10AD from the Harwell program library to numerically maximize this function, i.e., to calculate the FIML estimates. We note that the factor demand system (7) and (8) in conjunction with (2) and (3) may be viewed as a system of equations with implicitly defined variables.[5]

III. DATA AND PARAMETER ESTIMATES

We have estimated model (7) and (8) together with (2) and (3) using U.S. total manufacturing data for the period 1960–1988. The estimation requires data on gross output, labor and materials inputs, gross investment in plant and equipment capital and R&D capital and corresponding prices. (The capital stock data are generated internally by the model.)

The data on constant 1987 dollar gross output, labor, materials and corresponding prices were derived from the KLEMS data set prepared by the Division of Productivity Research of the Bureau of Labor Statistics, U.S. Department of Labor. For a description of the underlying methodology see, e.g., U.S. Department of Labor, Bureau of Labor Statistics [1983], Gullickson and Harper [1987], and Dean and Kunze [1988]. Our measure of material inputs is a Tornquist aggregate of the energy, materials and purchased service data in the KLEMS data set.[6]

The data on constant 1987 dollar gross investment in plant and equipment were obtained from the National Income and Wealth Division of the Bureau of Economic Analysis, U.S. Department of Commerce. For a description of the underlying methodology see, e.g., U.S. Department of Commerce, Bureau of Economic Analysis [1987] and Musgrave [1992]. Data on total (federal and company) nominal R&D investment are taken from National Science Foundation [1989] and earlier issues. The Jaffee-Griliches deflator for private nonfarm business reported in U.S. Department of Labor, Bureau of Labor Statistics [1989] is used as the deflator for R&D expenditures. We face a difficulty in choosing the initial values of the stocks of physical and R&D capital. For capital we set the initial stock in 1958 equal to the value of the constant 1987 dollar gross capital stock reported by Musgrave [1992]. For R&D we calculate the initial stock by dividing total constant dollar R&D expenditures by the growth rate of the gross capital stock reported in Musgrave [1992] plus 10 percent as an initial guess for the depreciation rate.[7] This calculation is motivated by the fact that $R_{t-1} = I_t^R / (g_t^R + \delta^R)$ where g_t^R is the growth rate of R_t. To avoid double counting we subtracted the labor and material components of R&D investment from the labor and material inputs.

In estimating the model all constant dollar variables were normalized by respective sample means. Prices were constructed conformably. To calculate expectations on output and the price of materials (normalized by the price of labor input) we first estimated a corresponding second-order vector autoregressive process. We then used this process to predict Y_t and p_t^M.[8]

5. Subroutine VA10AD calculates the gradient of the objective function numerically. For an algorithm for the computation of estimators of the parameters of a system of equations with implicitly defined variables that evaluates the gradient of the objective function from analytic expressions see, e.g., Prucha and Nadiri [1988].

6. We note that the adopted aggregation method is the same as that underlying the other aggregates in the KLEMS data set. One reason for using an aggregate measure of material inputs was to keep the analysis focused and to preserve degrees of freedom.

7. As discussed in more detail below, we find that our estimates for the depreciation rates in particular are quite insensitive to alternative choices for the initial stocks. As indicated above, in future work it may be of interest to estimate the initial stocks jointly with the other model parameters.

8. As discussed in more detail below, we also find that our estimates for the depreciation rates are quite insensitive to alternative choices for the expectation formation process.

TABLE I
Full Information Maximum Likelihood Estimates of the Parameters for the U.S. Total Manufacturing Sector: 1960–1988*

Parameter	Parameter Estimate	Standard Error
α_0	3.13	1.52
α_K	-13.15	3.08
α_R	1.60	1.75
α_{KK}	20.19	5.47
α_{RR}	2.94	3.69
α_{RK}	-6.34	4.39
α_M	0.41	0.19
α_{MM}	-0.10	0.08
α_{KM}	-1.45	1.24
αRM	-1.12	0.54
δ^K	0.059	0.008
δ^R	0.120	0.025
Log of likelihood	115.28	
M– Equation:	$R^2 = 0.98$,	DW = 1.90
L– Equation:	$R^2 = 0.73$,	DW = 1.77

*The R^2 values correspond to the squared correlation coefficients between the actual M and L variables and their fitted values.

Full information maximum likelihood estimates for the parameters are given in Table I.[9] The parameter estimates satisfy the theoretical restrictions. In particular, the estimate for α_{MM} is negative and that for α_{KK} is positive. Furthermore, α_{RR} and $\alpha_{KK}\alpha_{RR} - \alpha_{KR}^2$ are positive. Thus the estimated normalized variable cost function is concave in p^M and convex in K and R. The derivatives of the estimated variable cost function with respect to output are positive, and those with respect to the stocks of physical and R&D capital are negative.

9. We allow for first-order autocorrelation in the residuals. The estimated autocorrelation coefficients are close to unity. The reported standard errors are computed from a numerical estimate of the Hessian.

IV. EMPIRICAL RESULTS

Depreciation of Physical Capital

As discussed by Hulten [1991], the Bureau of Economic Analysis's capital stock studies are based on the perpetual inventory method. The Bureau uses constant estimates for service lives by type of assets and industry obtained from specific indus-

TABLE II
Estimates of the Depreciation Rates of Physical Capital Stocks

Source	Range of Estimates	Average Estimate
Musgrave [1992]	0.030–0.038	0.034
Epstein and Denny [1980]	0.108–0.138	0.126
Kollintzas and Choi [1985]	0.107–0.141	0.125
Bischoff and Kokkelenberg [1987]	0.096–0.118	0.106
This study		0.059

try studies. The service lives are designed to take account of expected average obsolescence over time and the normal deviations around the average life of the asset. Adjustments are made for retirement of assets at different ages by modifying the Winfrey [1935] S-3 curve, a bell-shaped distribution centered on the average life. The efficiency pattern for each subcohort of investment is calculated under the assumption of one-hoss-shay depreciation.

The average depreciation rate for the Bureau's gross capital stock estimates for plant and equipment developed by Musgrave [1992] for the U.S. total manufacturing sector between 1959 and 1988 is given in Table II. The depreciation rate estimates for total plant and equipment capital obtained in econometric studies using a factor demand modeling framework are listed also in Table II. These studies are based largely on a data set for the total U.S. manufacturing sector developed by Berndt and Wood [1975]. The sample period in the Epstein and Denny [1980] and Kollintzas and Choi [1985] studies is 1947–1971, while the Bischoff and Kokkelenberg [1987] study covers the period 1947–1978 and is based on quarterly data. The average values of the depreciation rates reported by these studies are quite similar. However in Epstein and Denny, the depreciation rates show a generally rising trend with some cycles, while the Kollintzas and Choi depreciation rates show an unbroken upward trend. Bischoff and Kokkelenberg's estimates show a different cyclical pattern than that reported by Epstein and Denny, but no trend. The average depreciation rates reported by these studies range between 10 and 12.5 percent, and are about three or four times as large as those reported by the Bureau of Economic Analysis.

Our estimates of the depreciation rate of gross capital are most comparable to those of the Bureau of Economic Analysis, which vary very little over time. We obtain, using our econometric model, a depreciation rate of 0.059, which is nearly twice as large as the Bureau's for the same period and about half the magnitude of the estimates reported by the other studies.[10] One possible explanation of the differences between our estimate and that of

10. To check the sensitivity of our results we reestimated the model under alternative assumptions. In particular, we estimated the model setting the initial stock of physical capital in 1948 (rather than in 1958) equal to the corresponding value of the Bureau of Economic Analysis's gross capital stock series, which yielded an estimate for δ of .068. Increasing or decreasing the initial stock for R&D capital by 10 percent left the estimate for δ essentially unchanged. When expectations are assumed to be formed from a first-order autoregessive process on Y with static expectations on p, or from a first-order vector autoregressive process on Y and p, we obtain 0.069 and 0.066 as an estimate for δ, respectively, but a somewhat smaller value for the log-likelihood.

the Bureau of Economic Analysis is that the latter does not take into account unexpected obsolescence due to changes in market conditions and changes in technology.[11] Unlike the Bureau's, our estimate of the depreciation rate is model driven. The point estimate could differ if the underlying model is reformulated or if demand equations for the quasi-fixed investments in physical and R&D capital are jointly estimated with the demand equations for the variable inputs, labor and materials. It seems of interest to check in future work the robustness of the results against alternative model specifications and also to allow for the disaggregation of the physical capital stock.

We next discuss the magnitude of the estimated depreciation rate as it relates to the average survival time of capital in more detail. According to the perpetual inventory method the stock of capital is in general calculated as

$$K_t = \sum_{i=0}^{\infty} \varphi_i I^K_{t-i}$$

where $\varphi_i \geq 0$ denotes the efficiency function. The φ_i are typically assumed to be nonincreasing, $\varphi_0 = 1$, $\varphi_i > 0$ for $i = 0,\dots, m$ and $\varphi_i = 0$ and $i > m$, where m is the maximal survival time (which may possibly be infinite). Given $K_t = I^K_t + (1 - \delta^K_t)K_{t-1}$ it follows that the depreciation rate can in general be expressed as

$$\delta^K_t = [\sum_{i=0}^{m}(\varphi_i - \varphi_{i+1})I^K_{t-i-1}] / [\sum_{i=0}^{m}\varphi_i I^K_{t-i-1}]. \tag{9}$$

11. Compare, e.g., Baily [1981] and Baily and Schultze [1990].

The average survival time is given by

$$\sum_{i=0}^{m}(\varphi_i - \varphi_{i+1})i.$$

Clearly, the depreciation rate will be constant if the φ_i decline geometrically, i.e., for $\varphi_i = (1 - \delta)^i$ we have $\delta^K_t = \delta$. That is, the depreciation rate is constant regardless of the pattern of investment. Another situation that yields a constant depreciation rate is the case where gross investment grows at a constant rate, i.e. $I^K_t = (1 + \rho_I)^t I^K_0$. In this situation the depreciation rate is constant over time and given by

$$\delta^K_t = [\sum_{i=0}^{m}(\varphi_i - \varphi_{i+1})(1 + \rho_I)^{-i}] / [\sum_{i=0}^{m}\varphi_i(1 + \rho_I)^{-i}]. \tag{10}$$

That is, the depreciation rate is only a function of $\varphi_0, \dots, \varphi_m$ and the growth rate of gross investment (and hence constant) regardless of the shape of the efficiency function. If investment grows rapidly, but not exactly exponentially, then δ^K_t will fluctuate, but may still be approximately constant.

We now use (10) and the average growth rate of investment to calculate estimates for the average survival time of physical capital for two "limiting" cases of efficiency functions. In the case of a one-hoss-shay efficiency function, i.e., $\varphi_i = 1$ for $i = 1, \dots, m$, the depreciation rate equals

$$\delta^K_t = 1 / [\sum_{i=0}^{m}(1 + \rho_I)^i],$$

and the average survival time equals the maximal survival time m. In the case of a geometrically declining efficiency function, i.e., $\varphi_i = (1 - \delta)^i$, the depreciation rate is constant regardless of the pattern of investment and is given by $\delta^K_t = \delta$, and the

TABLE III
Estimates of the Depreciation Rates of R&D Capital Stocks

Source	Range of Estimates	Average Estimate
Pakes and Schankerman [1978]	0.18–0.36	0.25
Pakes and Schankerman [1986]		
U.K.	0.17–0.26	
France	0.11	
Germany	0.11–0.12	
This study		0.12

average survival time equals $(1-\delta)/\delta$. The average growth rate of gross capital investment in our sample is 4 percent. Corresponding to this growth rate and our estimate of an average depreciation rate of 0.059, the implied average survival times for the geometrically declining and for the one-hoss-shay efficiency function are sixteen and thirteen years, respectively.

Depreciation of R&D Capital

It is well known that knowledge capital is a public good because it can be reproduced at very little or zero cost and because of appropriability problems. Because of the public goods nature of knowledge it is often argued that market incentives may create an underinvestment in knowledge producing activities (e.g., Arrow [1962]). The stock of knowledge is often approximated, albeit inadequately, by cumulating the R&D investment by firms and the public sector. To estimate whether there is a tendency for underinvestment in knowledge producing activities, the stock of R&D capital is often used as an input in the production function to estimate the private and social rates of return to R&D effort.

The estimates for both private and social rates of return in R&D investment have been very high in most industries (see Bernstein and Nadiri [1991]). The private rate of return in R&D investment is affected by the rate of decay of the private revenues accruing to industrially produced knowledge. However, except for the two studies by Pakes and Schankerman [1978; 1986] there are few estimates for the rate of decay of knowledge capital. Pakes and Schankerman correctly emphasize that the conceptually appropriate rate of depreciation of knowledge is the rate at which the appropriable revenues decline. The rate of decay in the revenues does not arise from any decay in productivity of knowledge but from reduction in market valuation, which arises due to inability to appropriate the benefits from the innovations and the obsolescence of original innovations by new ones.

Pakes and Schankerman employed data on patent renewal fees to estimate the decay rate for knowledge capital for several European countries. Their estimates are shown in Table III. In their first study, their reported point estimate for the rate of decay was about 0.25 with a 95 percent confidence interval between 0.18 and 0.36.

TABLE IV
The Growth Rates of Physical and R&D Capital Stocks

Period	BEA Capital Stock	Estimated Capital Stock	Estimated R&D Stock
1960–1988	0.035	0.025	0.025
1960–1969	0.039	0.024	0.034
1970–1979	0.040	0.030	0.007
1980–1988	0.025	0.021	0.036

The rate of decay even at the lower bound of the 95 percent confidence interval reported by Pakes and Schankerman is twice as large as the usual ad hoc measure of 10 percent for the depreciation rate often used in constructing the stock of R&D. In another study these authors report lower estimates of the depreciation rates for several of the countries. Their estimates suggest a very high decay rate of 0.26 for R&D capital in the U.K. in the 1950s which declines to about 0.17 in the period of 1960s and 1970s. The estimates for France and Germany are similar, about 0.12, and fairly stable over time. Even these reduced depreciation rates are still much larger than the decay rate generally assumed for physical capital.

The estimate of the depreciation rate δ^R we obtained is about 0.12, which is quite similar to the ad hoc assumption of the R&D depreciation rate used in many studies that use R&D capital stock as an input in the production function (Griliches [1980], Bernstein and Nadiri [1988; 1991], Mohnen, Nadiri and Prucha [1983; 1986], Nadiri and Prucha [1990a; 1990b].[12] Again, it should be pointed out that this point estimate may change if the basic underlying model is reformulated or when the evolution of the path of physical and R&D capital stocks are jointly estimated with the derived demand function for the variable inputs, labor and materials.

Using our estimate for the depreciation rate for R&D we can calculate the average survival time for R&D capital analogously to the approach taken for physical capital. The average growth rate of R&D investment in our sample is 5 percent. Corresponding to this growth rate and our estimate of an average depreciation rate of 0.12 suggests an average survival time of approximately seven years in the case of a geometric and one-hoss-shay efficiency function.

Physical and R&D Capital Stock

The model considered in this paper generated series for capital stocks and depreciation rates for physical and R&D capital as a byproduct of the estimation process. In Table IV we report the estimates for the rates of growth of these stocks for the entire sample period and several subperiods. For comparison purposes we also report the growth rate of the Bureau of Economic Analysis's capital stock series for the total manufacturing sector for the comparable periods. Because of the difference between the depreciation rate measured by the Bureau and our estimated depreciation rate, the Bureau's

12. As discussed above, we checked the sensitivity of our results by reestimating the model under alternative assumptions. Increasing or decreasing the initial stock of R&D capital by 10 percent yielded estimates for depreciation of 0.11 and 0.13, respectively. Setting the initial stock of physical capital in 1948 (rather than in 1958) equal to the corresponding value of the Bureau of Economic Analysis's gross capital stock series yielded an estimated depreciation rate of .14. Changing the expectations formation process left the estimated depreciation rate essentially unchanged.

TABLE V
The Ratios of Net and Replacement Investment to Gross Investment

Period	Physical Capital Investment		R&D Capital Investment	
	Net	Replacement	Net	Replacement
1960–1988	0.28	0.72	0.16	0.84
1960–1969	0.26	0.74	0.22	0.78
1970–1979	0.33	0.67	0.05	0.95
1980–1988	0.25	0.75	0.23	0.77

capital grows more rapidly than the capital stock generated internally by our model. The rates of growth of the physical capital stock measured by the Bureau of Economic Analysis are substantially larger than those generated by our model. This is particularly true for the period prior to 1980. As a result, at the end of the sample period, the two capital stock estimates diverge by approximately 28 percent.

The growth rate of the R&D capital stock shows considerable variation over the subperiods. The growth of the R&D stock collapses during the period 1970–79 when both private and federal real R&D investment remain fairly flat. It resumed its pre-1970 growth rate in the period 1980–88, in contrast to that of the physical capital stock, which decreased substantially in this period compared to the previous periods.

Composition of Gross Investment

Given our estimates for the depreciation rates of physical and R&D capital we can decompose the gross investment into net and replacement investment. Such a decomposition is important from a policy point of view. If the percentage of replacement investment to gross investment is very high the net capital accumulation in the economy is likely to be insufficient to support vigorous growth.

As pointed out by Jorgenson [1974] some of the previous studies on replacement investment were not fully consistent in that they employed capital stock data that were generated under a different set of assumptions than those maintained in those studies. We note that within our modeling framework the stocks of physical and R&D capital are internally generated by the model and hence our a decomposition is internally consistent.

Given our estimate of the gross stock in physical capital and the corresponding depreciation rate we decompose (observed) gross investment in physical capital as follows:

$$I_t^K = K_t - K_{t-1} + \delta^K K_{t-1}$$

where $K_t - K_{t-1}$ represents net investment and $\delta^K K_{t-1}$ represents replacement investment. The decomposition of gross investment in R&D capital I_t^R is defined analogously. In Table V we present the ratio of net investment and replacement investment to gross investment for both types of capital for the sample period and the subperiods. For the entire sample period net investment as a percent of gross investment for physical and R&D capital is about 28 and 16 percent. Replacement investment is the major component of gross investment for both physical and R&D capital, i.e., its share is about 0.72 and 0.84, respectively. This pattern generally holds for the subperiods as well, except for the 1970–79 period when the ratio of replacement investment for physical capital falls to 0.67 while that of R&D capital increases to very high rates of almost 0.95. These changes reflect the relatively high growth rate of gross investment in plant and equipment in this period and the collapse of the growth rate of R&D investment in the same period as noted above.

V. CONCLUSION AND SUGGESTIONS FOR FURTHER RESEARCH

In this paper we have specified and estimated a model of factor demand that allows for estimating jointly the depreciation rates of both physical and R&D capital for the U.S. total manufacturing sector. The main result of our study is that the depreciation rate for plant and equipment capital is 0.059 and for R&D capital is 0.12. Our estimate for the depreciation rate of physical capital is generally much lower than those reported by Epstein and Denny [1980], Bischoff and Kokkelenberg [1987] and Kollintzas and Choi [1985]. However, our estimate of the depreciation rate for plant and equipment gross capital is higher than the Bureau of Economic Analysis's estimate of (on average) 0.036.

As a consequence of the differences in the rate of depreciation for physical capital, the level of the capital stock generated by the model is at the end of the sample period about 28 percent lower than that estimated by the Bureau of Economic Analysis. Our depreciation rate for R&D capital is remarkably close to the ad hoc assumption typically used in constructing the stock of R&D by the perpetual inventory method.

Another finding of interest is the decomposition of gross investment in both types of capital using our estimates of the depreciation rates. The replacement investment for physical and R&D capital is, on average for the sample period, three to five times larger than the net investment. This pattern generally holds for the subperiods as well, except for the 1970–79 period, when the share of net investment in R&D to gross investment collapses to 0.05 from 0.22 for the period 1960–69. This situation reverses itself in the 1980s when we observe a sizable decline in the ratio of net investment in plant and equipment to gross investment. On the whole the low growth of gross investment in both R&D and physical capital has led net capital formation to be far from robust, thus failing to support a vigorous growth of the manufacturing sector.

The results presented here should be considered as preliminary. There are a number of issues that may affect the estimates of the depreciation rates we report. First, we imposed certain restrictions on the model such as the assumption of constant returns to scale and of constant rates of depreciation for the two types of capital. Clearly, the assumption of, a constant depreciation rate for physical capital is restrictive, given the aggregate nature of our investment data. Also, we have not incorporated at this stage demand equations for investment in R&D and physical capital as part of our estimating model. Furthermore, the robustness of the results needs to be checked against alternative functional forms for the restricted cost function. It would also be of interest to introduce a separate index of technical change in the model and estimate a measure of productivity growth. Some of these extensions were discussed in an earlier paper (Prucha and Nadiri [1991]). A further important extension of the model would be to introduce investment in equipment and structures separately, since the depreciation rates of these two types of capital are quite different.

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